

Prestack Depth Migration with Screen Propagators

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Outline

- Introduction
- Phase-Screen Propagators
- Wide-Angle Screen Propagators
- Imaging Conditions
- Numerical Examples
- Conclusions and Perspectives

Prestack Migration

Time domain methods:

- Finite-difference schemes
- Time domain imaging conditions
(Maybe quite accurate; Time consuming, Huge memory size)
*e.g. Whitmore & Lines, 1986;
Chang & McMechan, 1987, 1990*

Prestack Migration

Frequency/Wavenumber domain methods:

- Phase-shift-like schemes
- Frequency domain imaging conditions

(Maybe not very accurate; Efficient, Less memory size)

e.g. Gazdag, 1978; Gazdag & Sguazzero, 1984; Popovici, 1994

Screen Propagators

(Fourier Transform Based Propagators)

- Phase-screen propagators

Wu & Huang, 1992; Wu, Huang & Xie, 1995

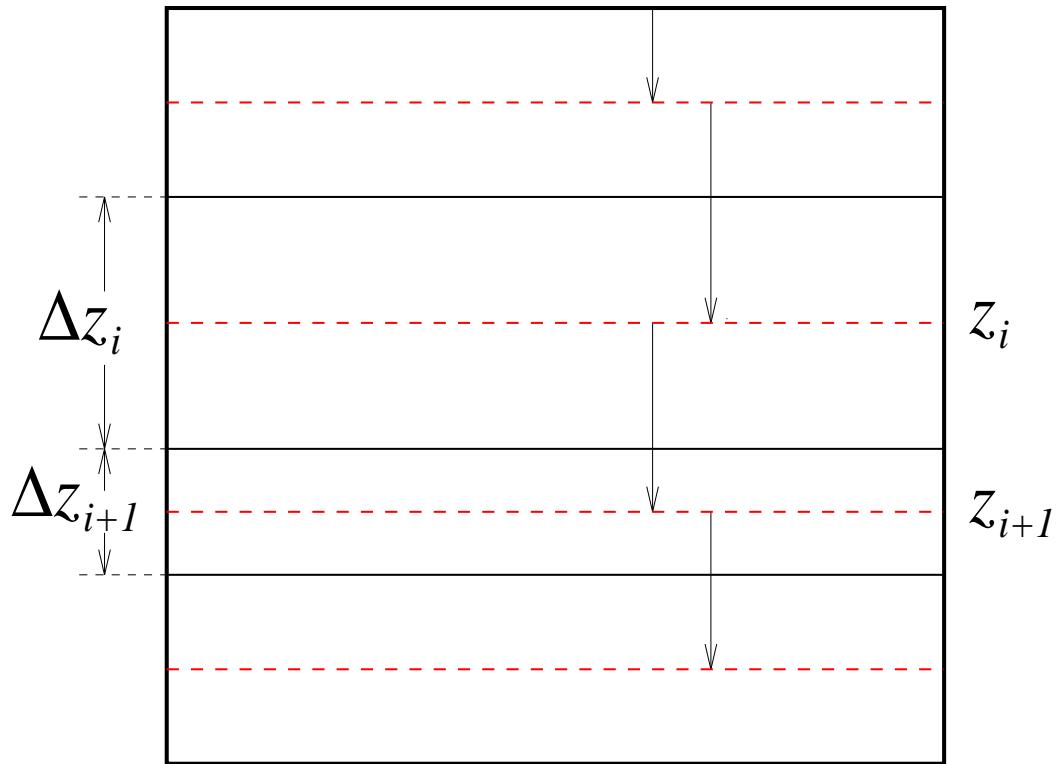
- Wide-angle screen propagators

Wu & Huang, 1995

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Phase-Screen Propagator for an acoustic constant-density medium



$$P(x, y, z_{i+1}; \omega) = \mathcal{F}_{k_x, k_y}^{-1} \left\{ \mathcal{F}_{x,y} \{ P(x, y, z_i; \omega) \} F_i^{i+1} \right\} H_{i+1}$$

$$F_i^{i+1} \equiv e^{\pm i(k_{z_i}\Delta z_i + k_{z_{i+1}}\Delta z_{i+1})/2}$$

$$H_{i+1} \equiv e^{\pm i\omega\Delta z_{i+1} \left[\frac{1}{v(x,y,z_{i+1})} - \frac{1}{v_0(z_{i+1})} \right]}$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2}$$

$$k = \frac{\omega}{v_0}$$

“+” for propagation from a source

“-” for backpropagation from a common source record

Phase-Screen Propagator for an acoustic non-constant-density medium

$$P(x, y, z_{i+1}; \omega) = \\ \mathcal{F}_{k_x, k_y}^{-1} \left\{ \mathcal{F}_{x, y} \{ P(x, y, z_i; \omega) \} F_i^{i+1} \right\} H_{i+1}$$

$$H_{i+1} \equiv e^{\pm i \frac{\omega}{v_0(z_{i+1})} (\varepsilon_{i+1}^\kappa - \varepsilon_{i+1}^\rho)}$$

$$\varepsilon_{i+1}^\kappa \equiv \left[\frac{\kappa_0(z_{i+1})}{\kappa(x, y, z_{i+1})} - 1 \right] \Delta z_{i+1}$$

$$\varepsilon_{i+1}^\rho \equiv \left[\frac{\rho_0(z_{i+1})}{\rho(x, y, z_{i+1})} - 1 \right] \Delta z_{i+1}$$

“+” for propagation from a source

“-” for backpropagation from a common source record

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Wide-Angle Screen Propagator for an acoustic constant-density medium

$$\begin{aligned}
 P(x, y, z_{i+1}; \omega) = & \\
 \mathcal{F}_{k_x, k_y}^{-1} \left\{ \mathcal{F}_{x, y} \left\{ P(x, y, z_i; \omega) \right\} F_i^{i+1} \right. & \\
 \left. + P_s(k_x, k_y, z_{i+1} \Delta z; \omega) \right\}
 \end{aligned}$$

$$\begin{aligned}
 P_s(k_x, k_y, z_{i+1}; \omega) = & \pm \frac{i}{2k_z} k^2 \\
 \mathcal{F}_{x, y} \left\{ \varepsilon_{i+1} \mathcal{F}_{k_x, k_y}^{-1} \left\{ \mathcal{F}_{x, y} \left\{ P(x, y, z_i; \omega) \right\} F_i^{i+1} \right\} \right\}
 \end{aligned}$$

$$\varepsilon_{i+1} \equiv \left[\frac{v_0^2(z_{i+1})}{v^2(x, y, z_{i+1})} - 1 \right] \Delta z_{i+1}$$

“+” for propagation from a source

“-” for backpropagation from a common source record

Wide-Angle Screen Propagator for an acoustic non-constant-density medium

$$\begin{aligned} P(x, y, z_{i+1}; \omega) = & \\ & \mathcal{F}_{k_x, k_y}^{-1} \left\{ \mathcal{F}_{x,y} \left\{ P(x, y, z_i; \omega) \right\} F_i^{i+1} \right. \\ & \left. + P_s(k_x, k_y, z_{i+1} \Delta z; \omega) \right\} \end{aligned}$$

$$\begin{aligned} P_s(k_x, k_y, z_{i+1}; \omega) = & \pm \frac{i}{2k_z} k^2 \\ & \left\{ \mathcal{F}_{x,y} \left\{ \varepsilon_{i+1}^\kappa P^0(x, y, z_{i+1}; \omega) \right\} \right. \\ & \left. + i \frac{\hat{k}}{k} \cdot \mathcal{F}_{x,y} \left\{ \varepsilon_{i+1}^\rho \nabla P^0(x, y, z_{i+1}; \omega) \right\} \right\} \end{aligned}$$

$$\begin{aligned} P^0(x, y, z_{i+1}; \omega) = & \mathcal{F}_{k_x, k_y}^{-1} \left\{ \mathcal{F}_{x,y} \left\{ P(x, y, z_i; \omega) \right\} F_i^{i+1} \right\} \\ \hat{k} = & \frac{1}{k} (k_x, k_y, k_z) \end{aligned}$$

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Imaging Conditions

$$M_S(x, y, z) = - \int i\omega P_+(x, y, z; \omega) P_-^*(x, y, z; \omega) d\omega$$

(cf Mittet et al. , 1995)

$$M_S(x, y, z) = - \int_{-\tau}^{\tau} \int i\omega P_+(x, y, z; \omega) e^{i\omega t} P_-^*(x, y, z; \omega) d\omega dt$$

$$M_S(x, y, z) = - \int i 2 \sin(\omega\tau) P_+(x, y, z; \omega) P_-^*(x, y, z; \omega) d\omega$$

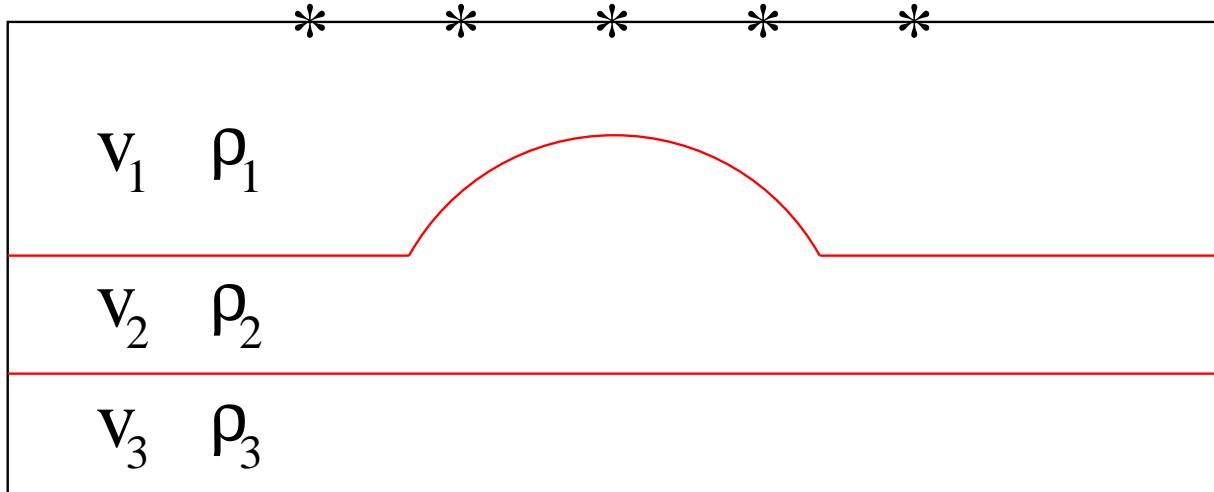
The final migrated image:

$$M(x, y, z) = \sum_{S=1}^{N_{shot}} M_S(x, y, z)$$

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Convex Model



Parameters for Modeling

$$N_t = 2048 \quad dt = 0.001\text{sec.}$$

$$N_x = 512 \quad N_z = 230 \quad \Delta x = \Delta z = 5m$$

$$N_{receiver} = 512$$

Ricker's wavelet: $f_p = 20Hz$

A FD scheme was used to generate synthetic data.

Parameters for Migration

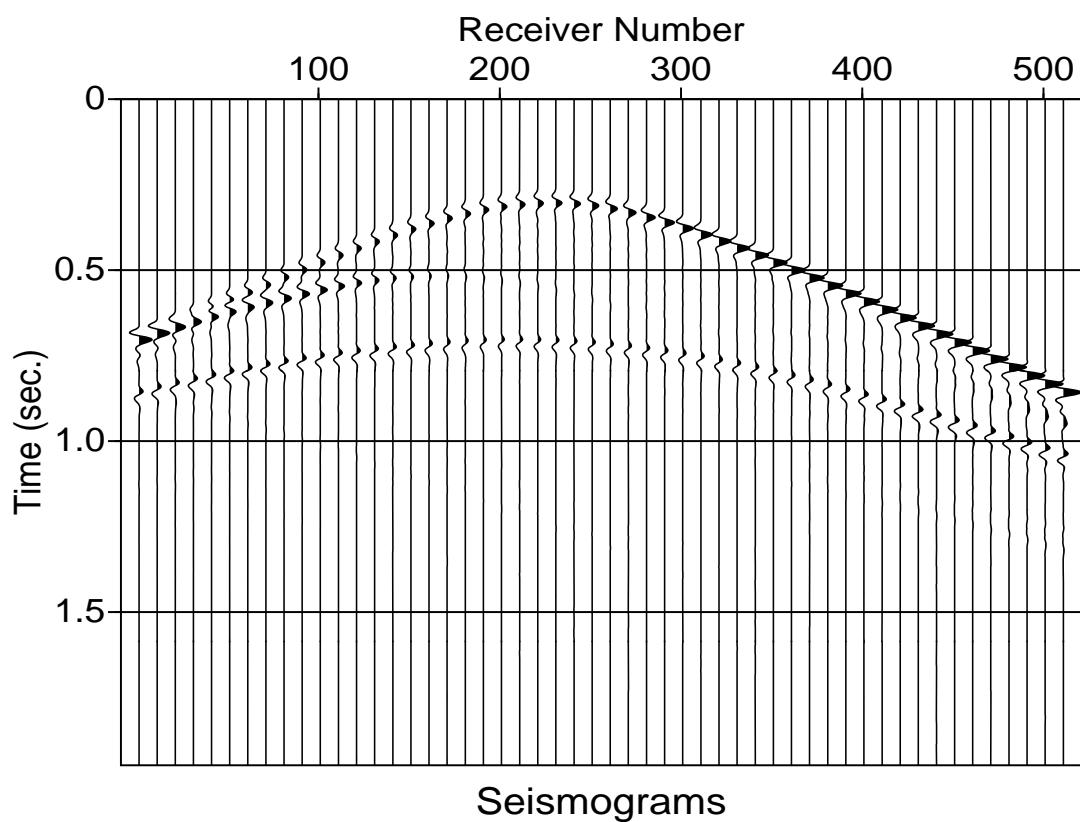
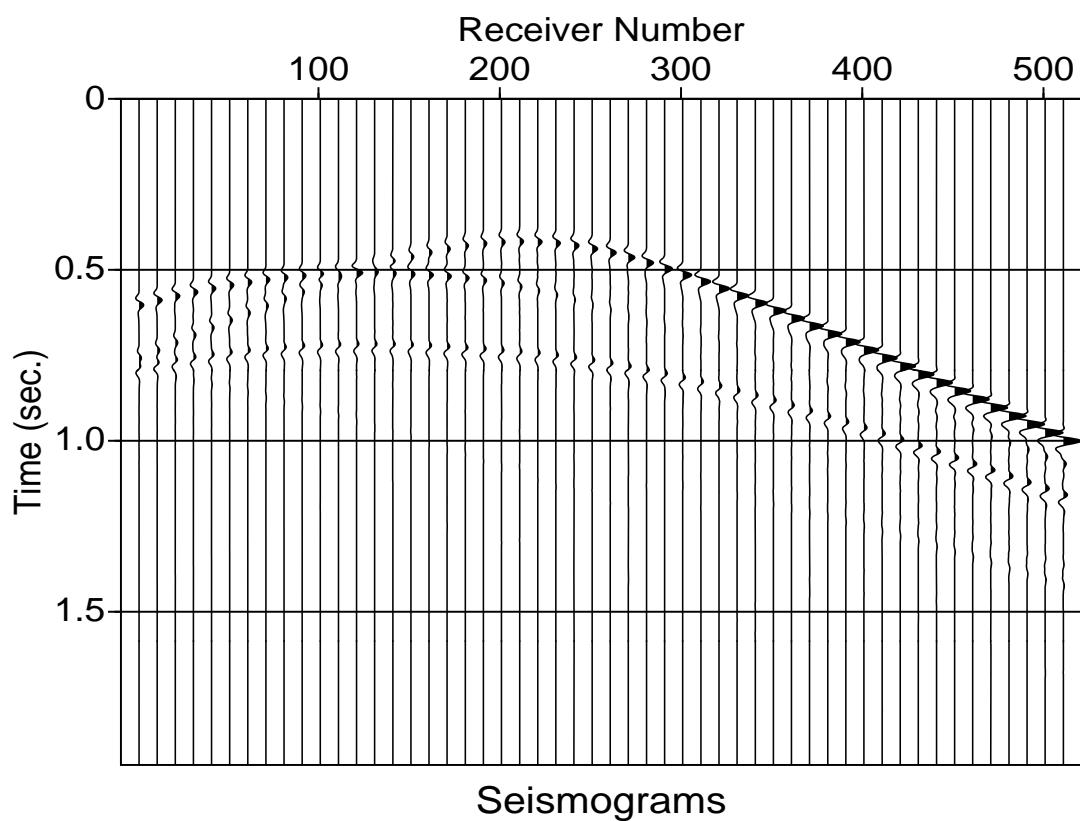
$$N_x = 512 \quad N_z = 200 \quad \Delta x = \Delta z = 5m$$

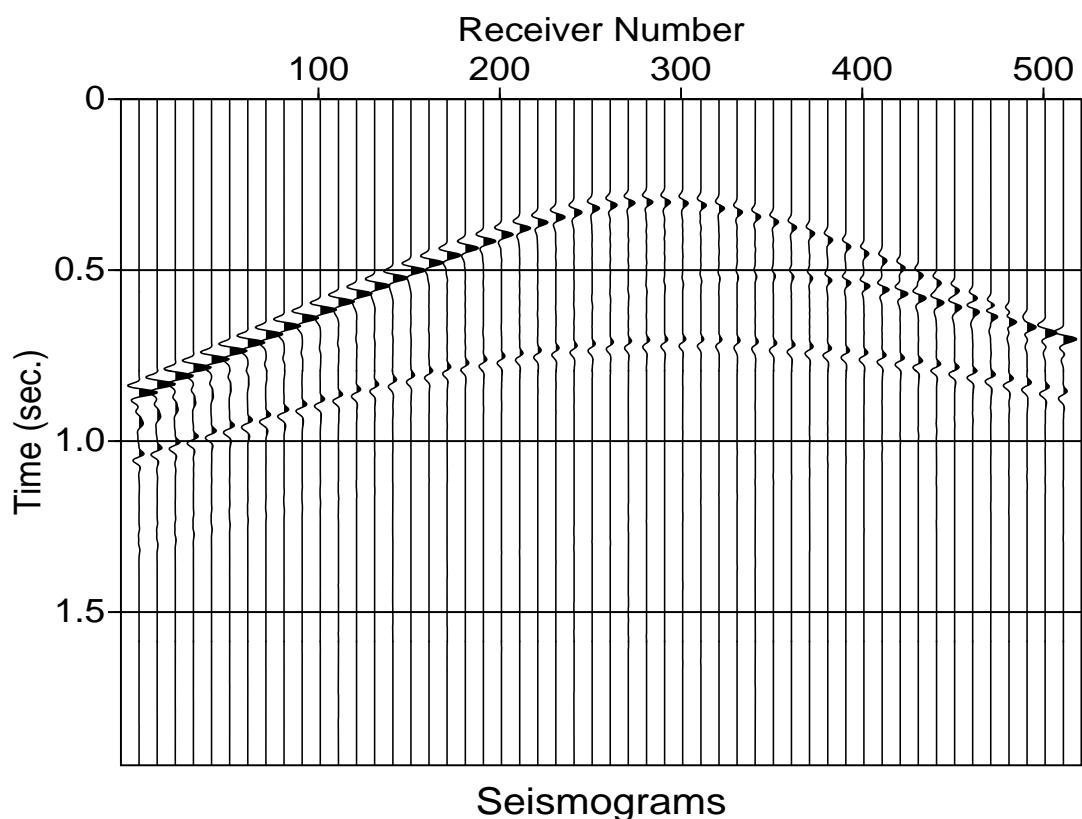
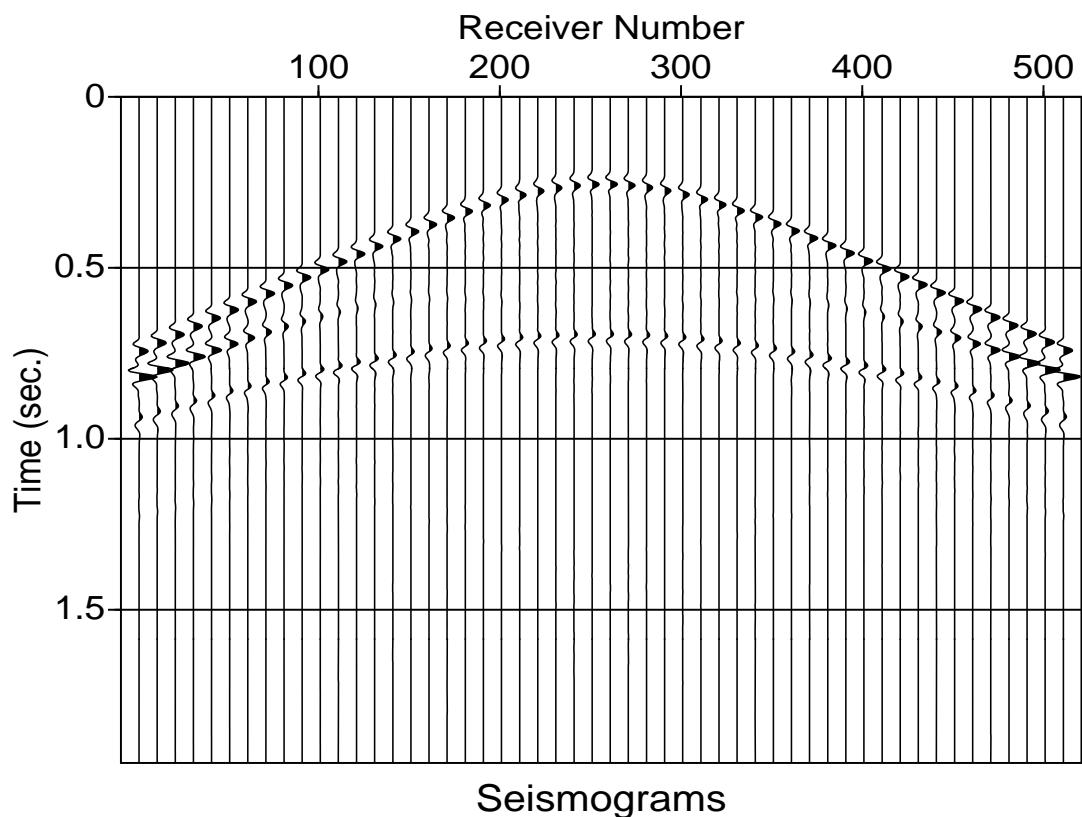
$$N_t = 512 \quad dt = 0.004\text{sec.}$$

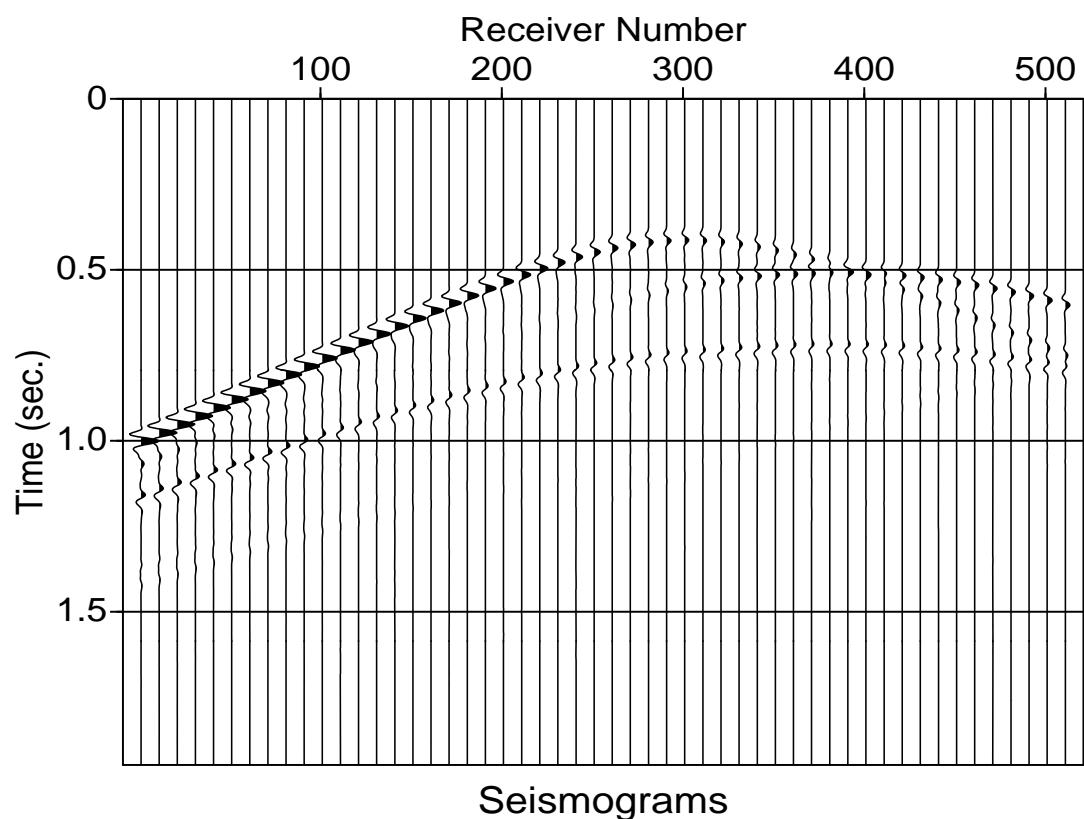
$f = 0.5 \sim 50Hz$ with 102 frequency components

$$N_{shot} = 5$$

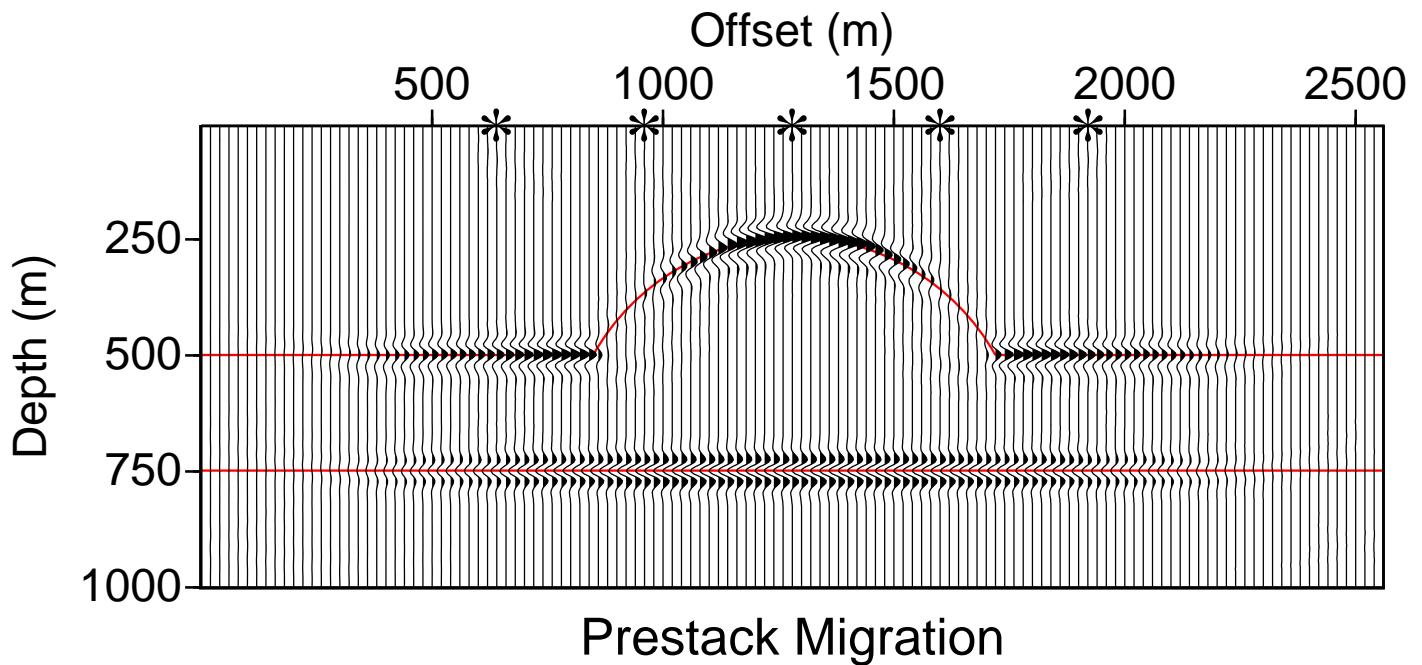
at $(128,1), (192,1), (256,1), (320,1), (384,1)$







Prestack Depth Migration with a Phase-Screen Propagator



$$v_1 = 2000 \text{ m/s}$$

$$\rho_1 = 1.0 \text{ g/cm}^3$$

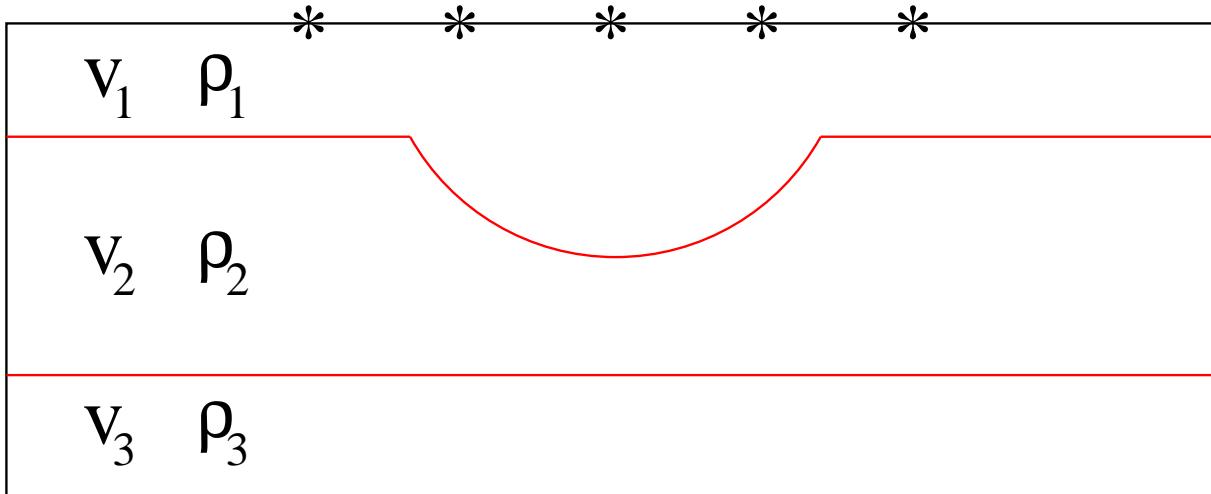
$$v_2 = 2200 \text{ m/s}$$

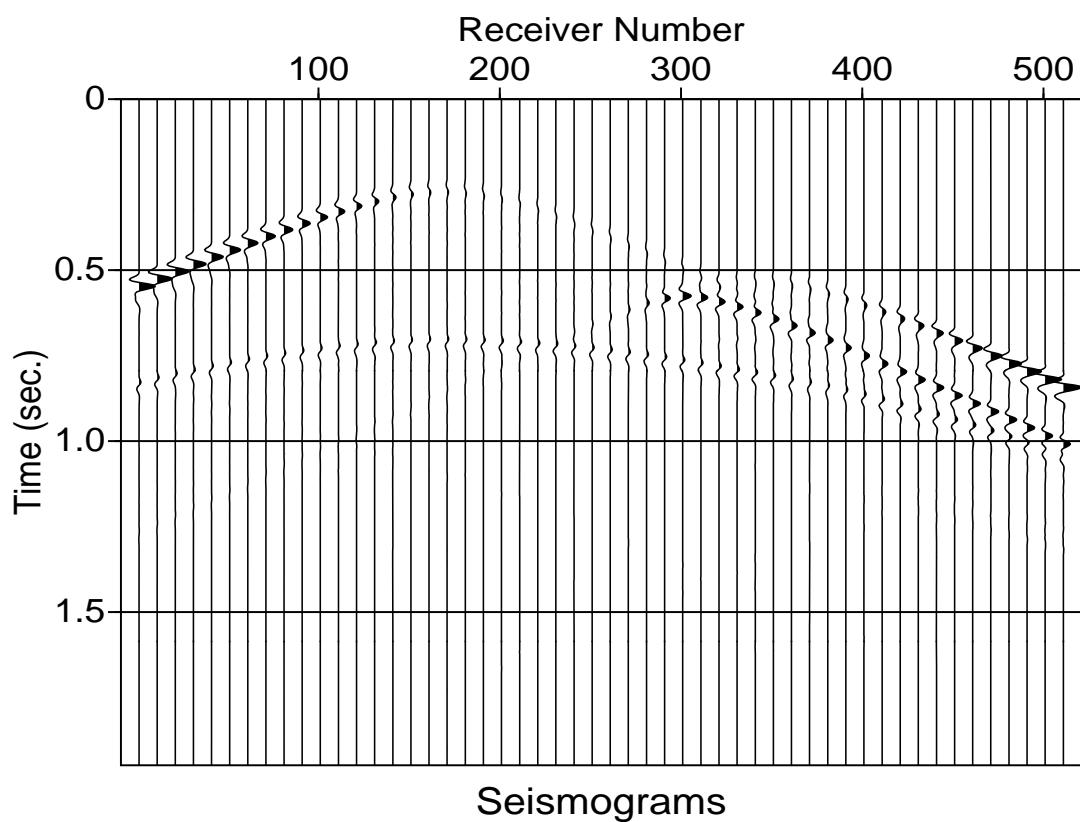
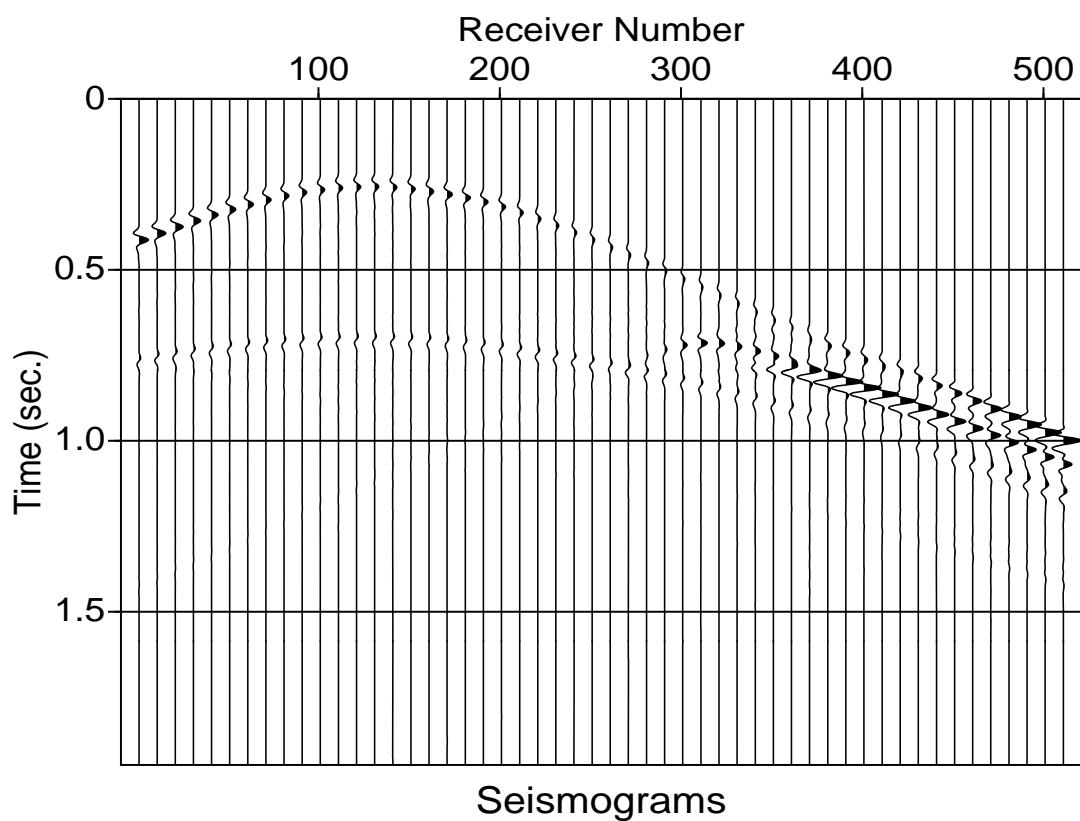
$$\rho_2 = 1.0 \text{ g/cm}^3$$

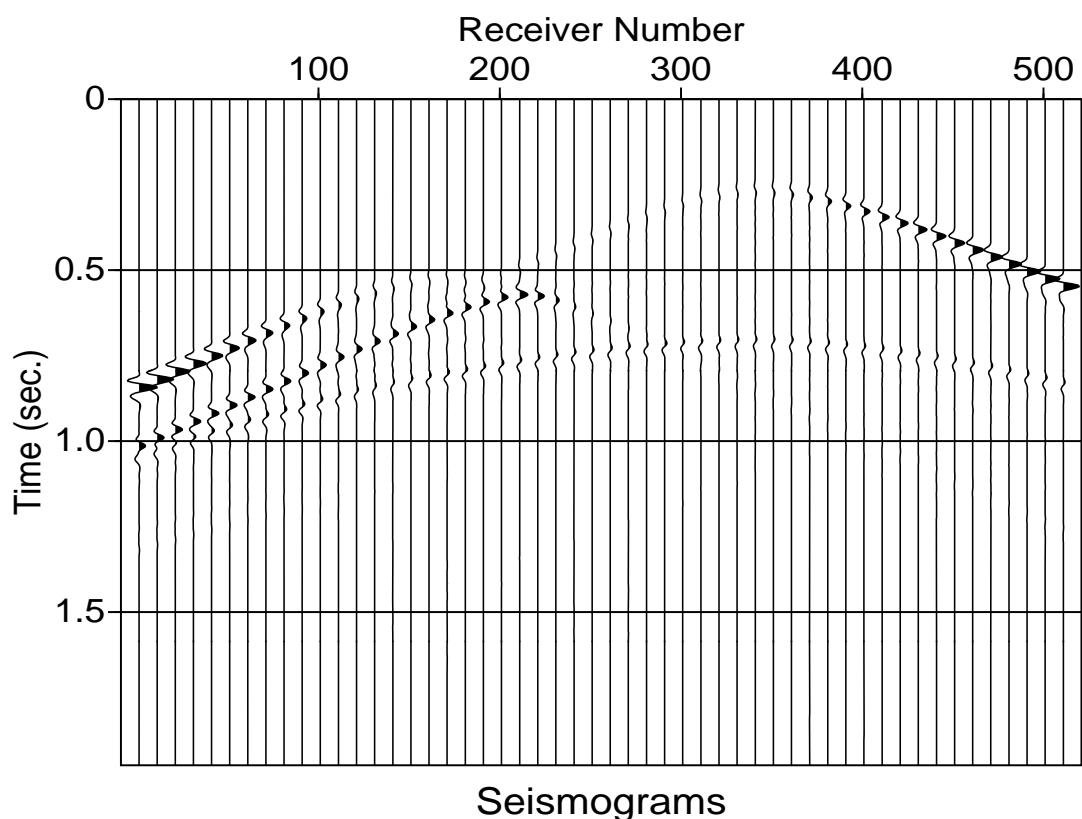
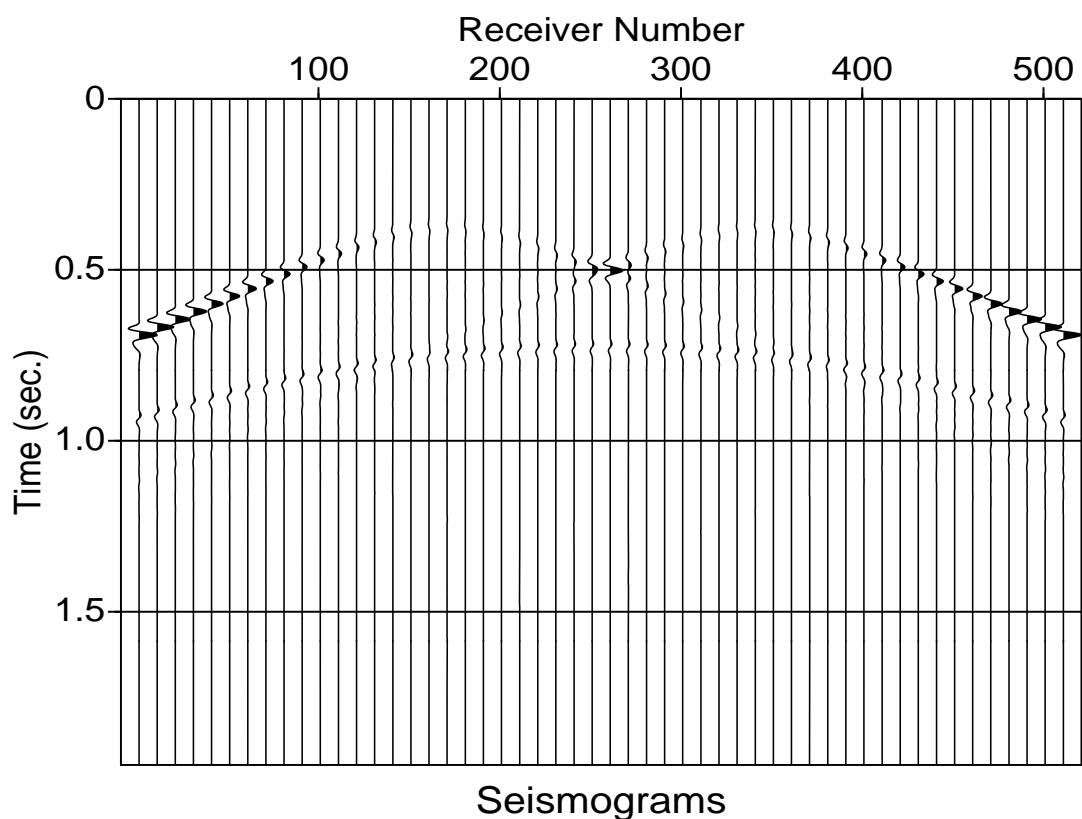
$$v_3 = 2000 \text{ m/s}$$

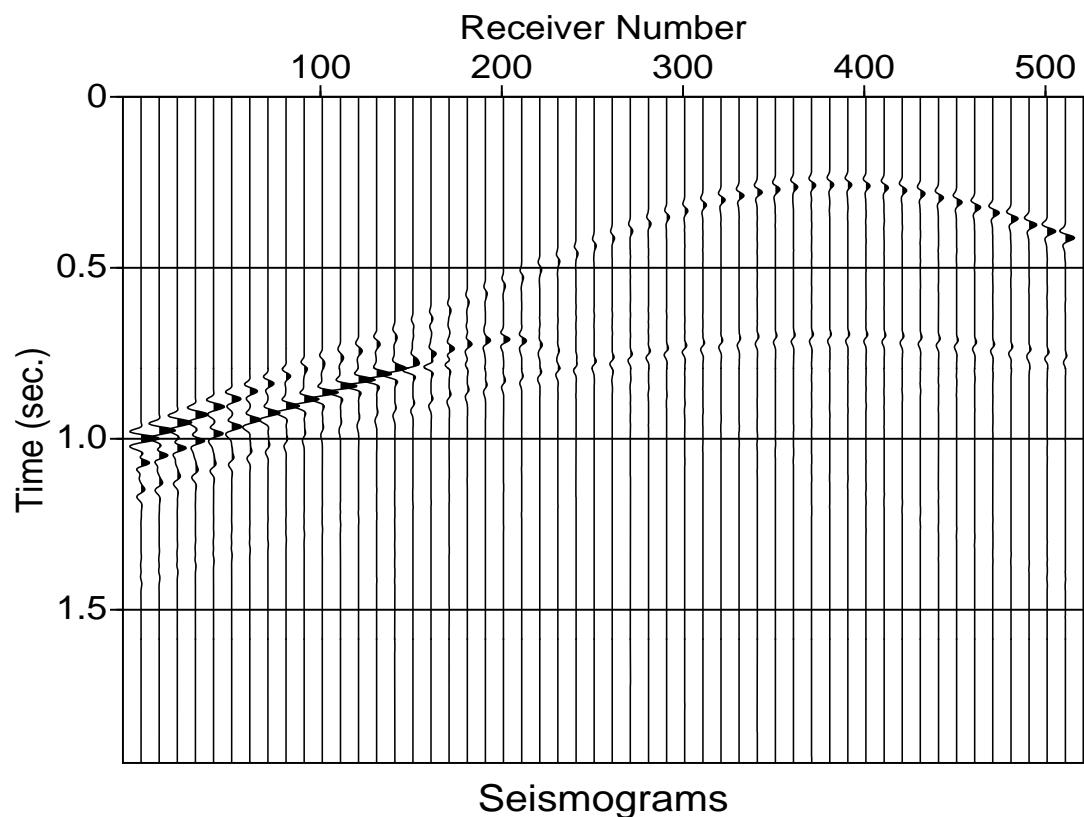
$$\rho_3 = 1.0 \text{ g/cm}^3$$

Concave Model

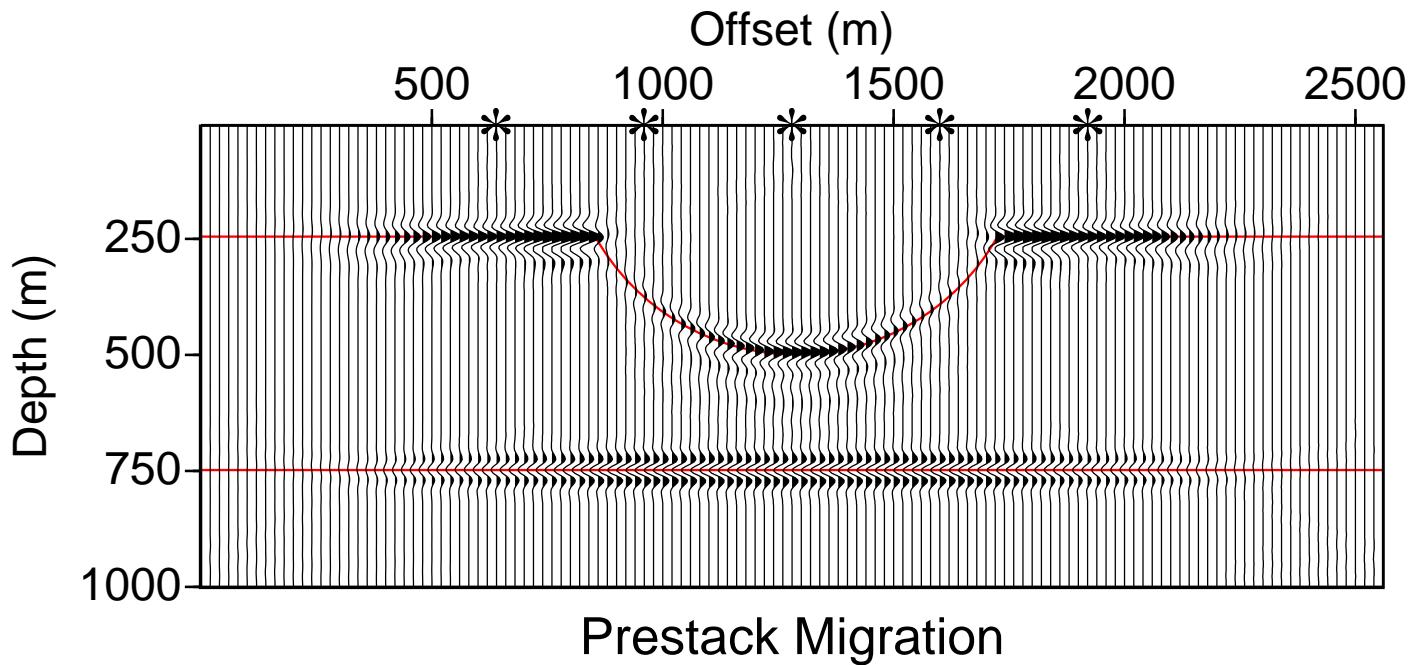








Prestack Depth Migration with a Phase-Screen Propagator



$$v_1 = 2000 \text{ m/s}$$

$$\rho_1 = 1.0 \text{ g/cm}^3$$

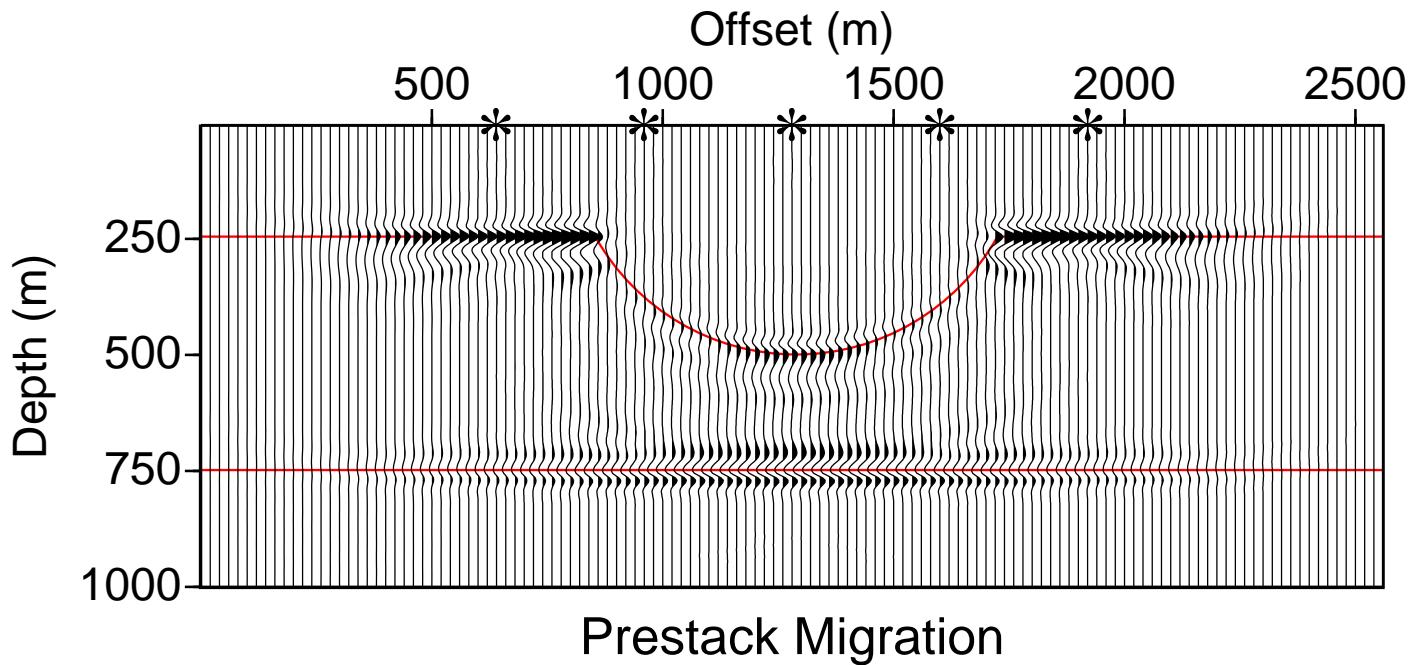
$$v_2 = 2200 \text{ m/s}$$

$$\rho_2 = 1.0 \text{ g/cm}^3$$

$$v_3 = 2000 \text{ m/s}$$

$$\rho_3 = 1.0 \text{ g/cm}^3$$

Prestack Depth Migration with a Phase-Screen Propagator



$$v_1 = 2000 \text{ m/s}$$

$$\rho_1 = 1.0 \text{ g/cm}^3$$

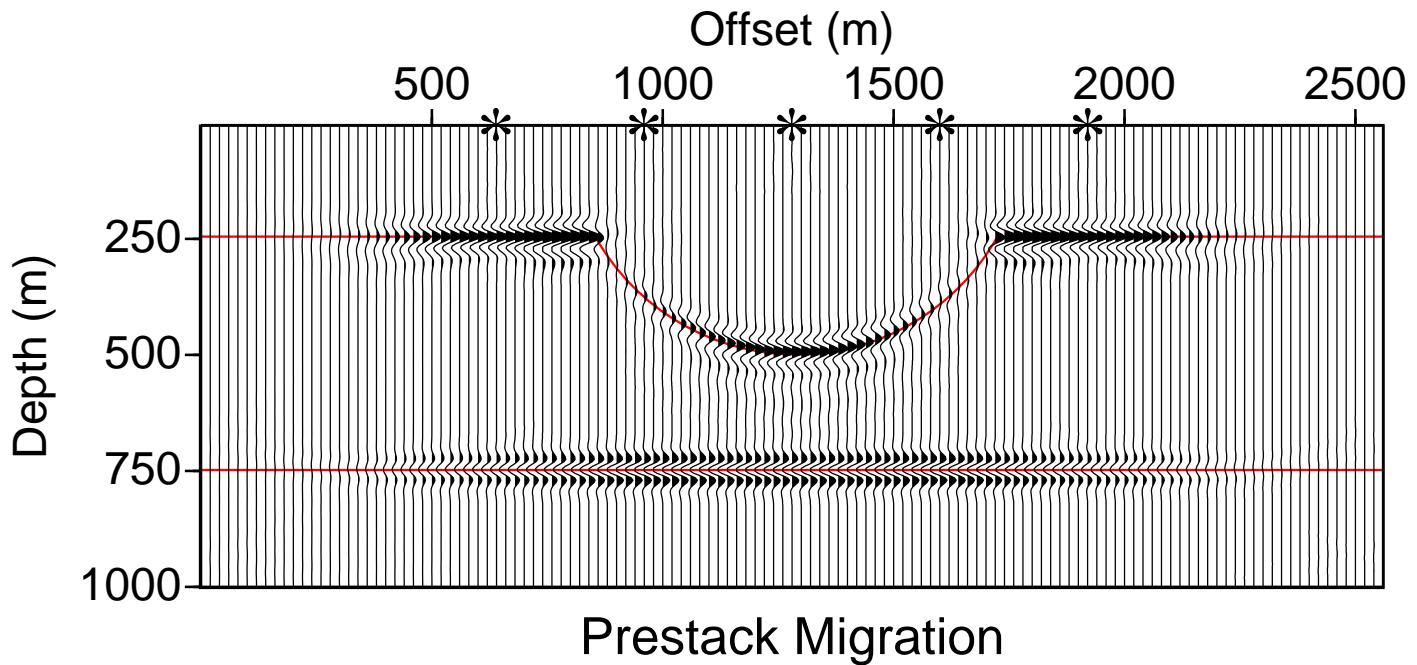
$$v_2 = 3000 \text{ m/s}$$

$$\rho_2 = 1.0 \text{ g/cm}^3$$

$$v_3 = 2000 \text{ m/s}$$

$$\rho_3 = 1.0 \text{ g/cm}^3$$

Prestack Depth Migration with a Phase-Screen Propagator



$$v_1 = 2000 \text{ m/s}$$

$$\rho_1 = 1.0 \text{ g/cm}^3$$

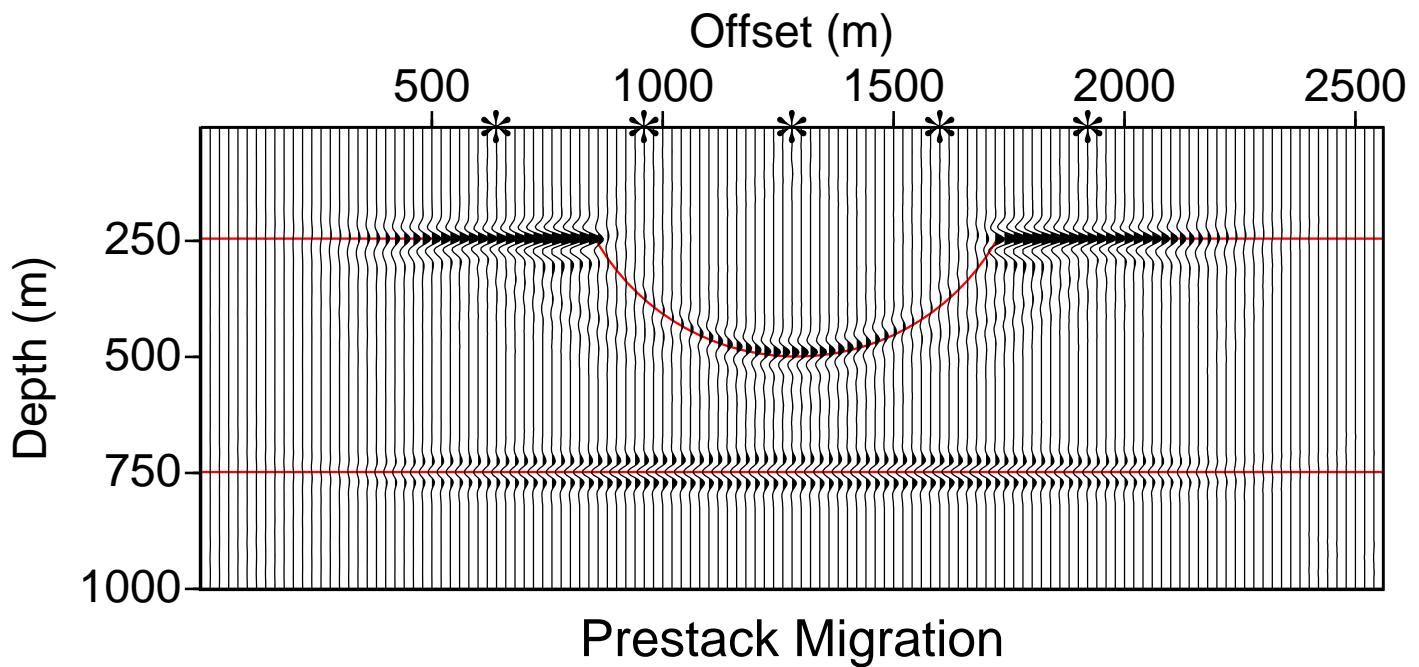
$$v_2 = 2200 \text{ m/s}$$

$$\rho_2 = 1.1 \text{ g/cm}^3$$

$$v_3 = 2000 \text{ m/s}$$

$$\rho_3 = 1.0 \text{ g/cm}^3$$

Prestack Depth Migration with a Wide-Angle Screen Propagator



$$v_1 = 2000 \text{ m/s}$$

$$\rho_1 = 1.0 \text{ g/cm}^3$$

$$v_2 = 2200 \text{ m/s}$$

$$\rho_2 = 1.0 \text{ g/cm}^3$$

$$v_3 = 2000 \text{ m/s}$$

$$\rho_3 = 1.0 \text{ g/cm}^3$$

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Conclusions and Perspectives

- Prestack migration with phase-screen propagators
- Prestack migration with wide-angle screen propagators
- More work on wide-angle cases
- 3-D
- Comparisons with other methods (accuracy and efficiency)